PreCalculus 11 Date _____

Unit 4: Quadratic Equations

4.0 Factoring Review

To factor means to write as a product.

- **Reverse of multiplying**

GCF Factoring (*always the first thing to look for)

ex. Factor each expression

a) $16r^2 - 20r$

$= 4r(4r - 5)$

b) $12n^5p^4 + 8n^4p^3 - 4n^3p^2$

$= 4n^3p^2(3n^2p^2 + 2np - 1)$

Group Factoring

ex. Factor each expression

a) $y^2 + 8xy + 2y + 16x$

$y(y + 8x) + 2(y + 8x)$

$(y + 8x)(y + 2)$

b) $a^2 - a + 3ab - 3b$

$a(a - 1) + 3b(a - 1)$

$(a - 1)(a + 3b)$

Decomposition Factoring ($ax^2 + bx + c, a \neq 1$)

ex. Factor each expression

a) $2x^2 + 7x - 4$

$\times$ -8

$+$ $+7$

$8, -1$

$2x^2 + 8x - x - 4$

$2x(x + 4) - 1(x + 4)$

$(x + 4)(2x - 1)$

b) $3m^2 + 2m - 5$

$\times$ -15

$+$ $+2$

$5, -3$

$3m^2 - 3m + 5m - 5$

$3m(m - 1) + 5(m - 1)$

$(m - 1)(3m + 5)$

Simple Trinomial Factoring ($x^2 + bx + c$) $a=1$

ex. Factor each expression

a) $x^2 - 3x - 10$

$(x)\ -10$
$(+)\ -3$
$-5, +2$

$(x-5)(x+2)$

b) $b^2 - 10b + 24$

$(x)\ 24$
$(+)\ -10$
$-4, -6$

$(b-4)(b-6)$

$b^4 - 10a^2b^2 + 24a^6$
$(b^2 - 4a^3)(b^2 - 6a^3)$

$b^6 - 10b^3 + 24$
$(b^3 - 4)(b^3 - 6)$

Differece of Squares ($a^2 - b^2$) $= (a-b)(a+b)$ conjugate pair

ex. Factor each expression

a) $x^2 - 81$

$(x-9)(x+9)$

b) $25a^2 - 16c^2$

$(5a + 4c)(5a - 4c)$

$25a^6 - 16c^{14}$
$(5a^3 + 4c^7)(5a^3 - 4c^7)$

Combinations

ex. Factor each expression

a) $-3a^3b - 51a^2b - 30ab$

$-3ab(a^2 + 17a + 10)$

$(x)\ +10$
$(+)\ +17$

b) $30x^2y - 25xy^2 - 30y^3$

$5y(6x^2 - 5xy - 6y^2)$

$(x)\ -36$
$(+)\ -5$
$-9, +4$

$6x^2 - 9xy + 4xy - 6y^2$

$3x(2x - 3y) + 2y(2x - 3y)$

$5y(2x-3y)(3x+2y)$

c) $7g^3h^2 - 28g^5$

$7g^3(h^2 - 4g^2)$

$7g^3(h-2g)(h+2g)$

d) $m^4 - 3m^2 - 4$

$(m^2 - 4)(m^2 + 1)$

$(x)\ -4$
$(+)\ -3$
$-4, +1$

$(m+2)(m-2)(m^2+1)$

Practice: WS4.0A – work on separate paper

PreCalculus 11 Date _____

Unit 3: Quadratic Equations

3.1 Graphical Solutions of Quadratic Equations

Quadratic Equation: a 2nd degree equation with standard form $ax^2+bx+c=0 \; ; a \neq 0$

ex. $2x^2+12x+16=0$

$3x^2 = -2+5x$

Root(s) of an equation:

- the solution(s) to an equation
- value(s) of the variable that satisfies the equation

Zero(s) of a function:

- the value(s) of $f(x)$ for which $f(x)=0$
- the x-intercept(s) of the graph of $f(x)$

ex. Determine the roots of each equation by graphing by hand:

$x^2-8x+16=0$

graph $f(x)=x^2-8x+16$

$=(x^2-8x+16)$ ~~$+16-16$~~

$f(x)=(x-4)^2$

vertex $(4,0)$

The zero(s) are the roots of the equation.

$x=4$

$-2x^2=8x$

$-2x^2-8x=0$ OR $0=2x^2+8x$

$f(x)=2(x^2+4x+4)-8$

$=2(x+2)^2-8$

$x=0,-4$

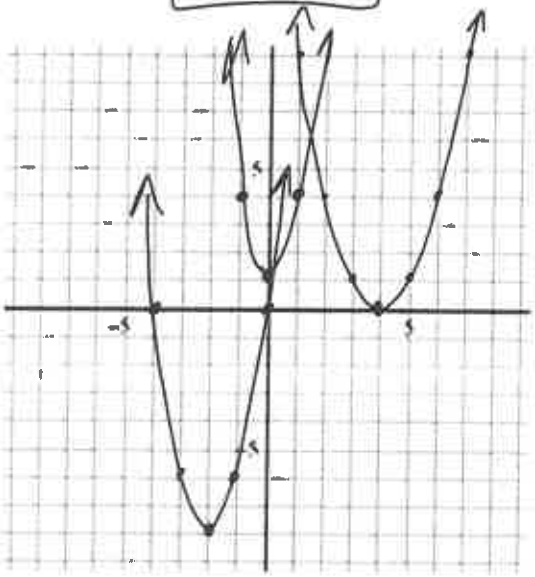
$3x^2=-1$

$3x^2+1=0$

$f(x)=3x^2+1$

no zeros

$\therefore$ no roots



Check:

$x^2-8x+16=0$

$(4)^2-8(4)+16$

$16-32+16$

$0=0$ ✓

$-2x^2=8x$

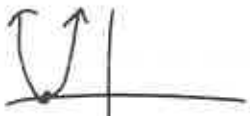
$-2(0)^2=8(0)$

$0=0$ ✓

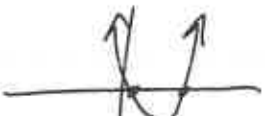
$-2(-4)^2=8(-4)$

$-32=-32$ ✓

Possible number of solutions to a quadratic equation:



1 real x-intercept
1 real zero
1 real root



2 real x-intercepts
2 real zeros
2 distinct real roots

no real x-intercepts
no real zeros
no real roots

ex. The manager of Jasmine's Fine Fashions is investigating the effect that raising or lowering dress prices has on the daily revenue from dress sales. The function $R(x) = 100 + 15x - x^2$ gives the store's revenue *R*, in dollars, from dress sales, where *x* is the price change, in dollars. What price changes will result in no revenue?

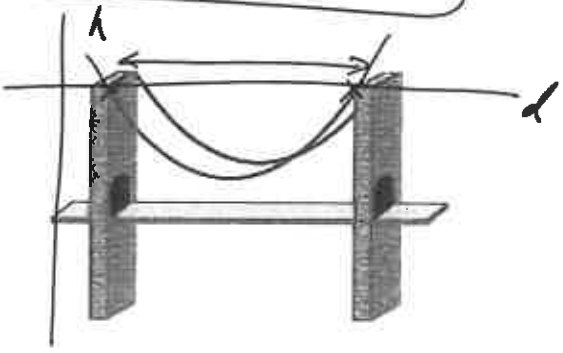
$R(x) = 0$

$100 + 15x - x^2 = 0$

$x = 20$ & $x = -5$

A decrease in price of $5 or an increase in price of $20 will result in no revenue.

ex. The curve of a suspension bridge cable attached between the tops of two towers can be modeled by the function $h(d) = 0.0025(d - 100)^2 - 10$, where *h* is the vertical distance from the top of a tower to the cable and *d* is the horizontal distance from the left end of the bridge, both in meters. What is the horizontal distance between the two towers? Express your answer to the nearest tenth of a meter.



$h(d) = 0$

$0.0025(d-100)^2 - 10 = 0$

$d = 36.75$ & $d = 163.25$

$163.25 - 36.75 =$ 126.5 m

Practice: pg. 215/ #1 – 3, 5 – 8, 10, 12, 13, 16 - 18

PreCalculus 11 Date _____

Unit 4: Quadratic Equations

4.2 Factoring Quadratic Equations

Zero Product Property: if the product of two numbers is zero then one or both of the numbers must be zero

ie. $a \cdot b = 0$ then $a = 0$ or $b = 0$

ex. Determine the roots of $(3x - 5)(x + 1) = 0$

$\therefore$ $3x - 5 = 0$ or $x + 1 = 0$

$x = \frac{5}{3}$ $\quad x = -1$

ex. Determine the roots of each quadratic equation.

$x^2 + 6x = -9$

$x^2 + 6x + 9 = 0$

$(x+3)(x+3) = 0$

$x + 3 = 0$ or $x + 3 = 0$

$x = -3$ $\quad x = -3$

$x^2 = 21 - 4x$

$x^2 + 4x - 21 = 0$

$(x+7)(x-3) = 0$

$x + 7 = 0$ or $x - 3 = 0$

$x = -7, 3$

$2x^2 - 9x - 5 = 0$

$2x^2 + x - 10x - 5 = 0$

$x(2x+1) - 5(2x+1) = 0$

$(2x+1)(x-5) = 0$

$2x + 1 = 0$ or $x - 5 = 0$

$x = -\frac{1}{2}, 5$

$2n^2 = n$

$2n^2 - n = 0$

$n(2n-1) = 0$

$n = 0$ or $2n - 1 = 0$

$n = 0, \frac{1}{2}$

$5b^2 - 45 = 0$ → OR square root property (no degree 1 term)

$5(b^2 - 9) = 0$

$5(b+3)(b-3) = 0$

$5 \neq 0$ $\quad b + 3 = 0$ $\quad b - 3 = 0$

$b = -3, +3$

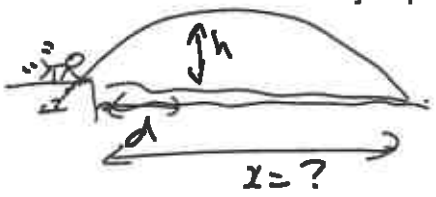
$5b^2 - 45 = 0$

$5b^2 = 45$

$\sqrt{b^2} = \pm\sqrt{9}$

$b = \pm 3$

ex. Dock jumping is an exciting dog event in which dogs compete for the longest jumping distance from a dock into a body of water. The path of a dog on a particular jump can be approximated by the quadratic function $h(d) = -\frac{3}{10}d^2 + \frac{11}{10}d + 2$, where h is the height above the surface of the water and d is the horizontal distance the dog travels from the base of the dock, both in feet. Determine the horizontal distance of the jump.



$h(d) = 0$

$10\left(-\frac{3}{10}d^2 + \frac{11}{10}d + 2\right) = (0)10$ * eliminate denominators by multiplying all terms by the LCD

$\rightarrow -3d^2 + 11d + 20 = 0$

$\rightarrow 3d^2 - 11d - 20 = 0$

⊗ -60, ⊕ -11: $-15, 4$

$3d^2 - 15d + 4d - 20 = 0$

$3d(d-5) + 4(d-5) = 0$

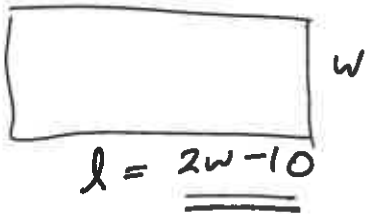
$(d-5)(3d+4) = 0$

$d - 5 = 0$ $\quad 3d + 4 = 0$

$d = 5$ $\quad d = -\frac{4}{3}$ reject

5 ft

ex. The length of an outdoor lacrosse field is 10 m less than twice the width. The area of the field is 6600 m^2. Determine the dimensions of an outdoor lacrosse field.



$w(2w-10) = 6600$

$2w^2 - 10w - 6600 = 0$

$2(w^2 - 5w - 3300) = 0$

$(w+55)(w-60) = 0$

$w + 55 = 0$ $\quad w - 60 = 0$

$w = -55$ $\quad w = 60$

60m by 110m

Practice: pg. 230/ # 7adf, 8abe, 9acd, 10de, 11 – 17, 18ab, 19, 20, 24, 30

PreCalculus 11 Date _____

Unit 4: Quadratic Equations

4.3 Solving Quadratic Equations by Completing the Square

Recall from 4.2 – Square Root Property (for quadratics with no degree 1 term)

ex. solve $2x^2 - 1 = 17$

$2x^2 =$

$\sqrt{x^2} = \sqrt{9}$

$x =$ ±

Extend to binomial square: **ex.** solve $(x-1)^2 - 49 = 0$

$\sqrt{(x-1)^2} = \pm\sqrt{49}$

$x - 1 = \pm 7$

$x = \pm 7 + 1$

$7 + 1 \qquad -7 + 1$

$8 \qquad -6$

If there is no square we can introduce one by completing the square.

Case I: $a = 1$

ex. Solve by completing the square. Give answers in exact form and as decimals rounded to nearest tenth if needed.

$x^2 - 21 = -10x$

$x^2 + 10x - 21 = 0$

$(x^2 + 10x + 25) - 21 - 25 = 0$

$(x+5)^2 - 46 = 0$

$(x+5)^2 = 46$

$x + 5 = \pm\sqrt{46}$

$x = -5 \pm \sqrt{46}$ Exact

$-5 + \sqrt{46} \qquad -5 - \sqrt{46}$

$= 1.8 \qquad = -11.8$

$q^2 - 4q = 14$

$(q^2 - 4q + 4) = 14 + 4$

$(q-2)^2 = 18$

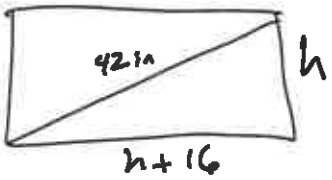
$q - 2 = \pm\sqrt{18}$

$\sqrt{9}\sqrt{2}$

$q = 2 \pm 3\sqrt{2}$

$q = 6.2, -2.2$

ex. A wide-screen television has a diagonal measure of 42 in. The width of the screen is 16 in. more than the height. Determine the dimensions of the screen, to the nearest tenth of an inch.



$h^2 + (h+16)^2 = 42^2$

$h^2 + h^2 + 32h + 256 = 1764$

$\frac{2h^2}{2} + \frac{32h}{2} = \frac{1508}{2}$

$h^2 + 16h + 64 = 754 + 64$

$(h+8)^2 = 818$

$h + 8 = \pm\sqrt{818}$

$h = -8 \pm \sqrt{818}$

$h = 20.6$ $\quad h = -36.6$ Extraneous root

20.6 m by 36.6 in

Case II: $a \neq 1$

ex. Solve by completing the square. Give answers in exact form and as decimals rounded to nearest hundredth if needed.

$\frac{4b^2 - 40b}{4} = \frac{-25}{4}$

$b^2 - 10b + 25 = -\frac{25}{4} + 25$

$(b-5)^2 = 75/4$

$b - 5 = \pm\sqrt{75/4}$

$b - 5 = \pm\frac{5\sqrt{3}}{2}$

$b = 5 \pm \frac{5\sqrt{3}}{2}$

$b = 9.33, 0.67$

$\frac{-2x^2 - 3x + 7 = 0}{-2}$

$x^2 + 3/2\,x + 9/16 = 56/16 + 9/16$

$(x + 3/4)^2 = 65/16$

$x + 3/4 = \pm\frac{\sqrt{65}}{4}$

$x = -3/4 \pm \frac{\sqrt{65}}{4}$

$x = \frac{-3 \pm \sqrt{65}}{4}$

$x = 1.27, -2.77$

ex. A defender kicks a soccer ball away from her own goal. The path of the kicked soccer ball can be approximated by the quadratic function $h(x) = -0.06x^2 + 3.168x - 35.34$, where x is the horizontal distance travelled, in metres, from the goal line and h is the height, in metres.

a) How far is the soccer ball from the goal line when it is kicked? Express your answer to the nearest tenth of a metre.

$-0.06x^2 + 3.168x - 35.34 = 0$ ball starts $h = 0$

$\frac{}{-0.06}$

$x^2 - 52.8x + 696.96 = -589 + 696.96$

$(x - 26.4)^2 = 107.96$

$x - 26.4 = \pm\sqrt{107.96}$

$x = 26.4 \pm \sqrt{107.96}$

$x = 36.8$ lands, 16.0

16.0 m from goal line

b) How far does the soccer ball travel before it hits the ground?

$36.8 - 16.0$

20.8 m

Practice: pg. 241/ #5acdf, 6, 7abe, 8, 10, 11 – 13, 15, 21

Quiz Tomorrow 4.1/4.2

PreCalculus 11 Date _____

Unit 4: Quadratic Equations

4.4 The Quadratic Formula

The Quadratic Formula: The roots of $ax^2+bx+c=0$ are given by

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

* on formula sheet

ex. Solve using the quadratic formula:

$3x^2 + 5x - 2 = 0$

$a=3 \quad b=5 \quad c=-2$

$$x = \frac{-5 \pm \sqrt{5^2-4(3)(-2)}}{2(3)}$$

$$= \frac{-5 \pm \sqrt{49}}{6}$$

$$= \frac{-5 \pm 7}{6} \qquad \frac{-5+7}{6} \text{ or } \frac{-5-7}{6}$$

$$x = \frac{1}{3} \text{ or } -2$$

$9x^2 + 4 = -12x$

$9x^2 + 12x + 4 = 0$

$a=9 \quad b=12 \quad c=4$

$$x = \frac{-12 \pm \sqrt{12^2-4(9)(4)}}{2(9)}$$

$$= \frac{-12 \pm \sqrt{144-144}}{18}$$

$$= \frac{-12 \pm 0}{18}$$

$$= -12/18 = \boxed{-2/3}$$

Discriminant:

$D = b^2-4ac$

The value of the discriminant (D) determines the nature of the roots

$D > 0$ 2 distinct real roots

$D = 0$ 1 distinct real root (2 equal real roots)

$D < 0$ no real roots

ex. Use the discriminant to determine the nature of the roots for each quadratic. Verify by graphing.

$3x^2 - 5x = -9$

$3x^2 - 5x + 9 = 0$

$D = b^2-4ac$

$= (-5)^2 - 4(3)(9)$

$= 25 - 108$

$= -83 < 0$

∴ no real roots

$4\left(\frac{x^2}{4}\right) = (3x-9)4$

$x^2 = 12x - 36$

$x^2 - 12x + 36 = 0$

$D = b^2-4ac$

$= (12)^2 - 4(1)(36)$

$= 144 - 144$

$= 0$

∴ 1 real root

ex. Solve using the quadratic formula. Answers in exact form and as decimals rounded to the nearest hundredth.

$3x^2 - 8x + 2 = 0$

$a=3 \quad b=-8 \quad c=2$

$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(3)(2)}}{2(3)}$$

$$= \frac{8 \pm \sqrt{64 - 24}}{6} \quad (40)$$

$$= \frac{8 \pm 2\sqrt{10}}{6}$$

$$= \boxed{\frac{4 \pm \sqrt{10}}{3}} \approx 2.39 \approx 0.28$$

$2x^2 + 3x = 1$

$2x^2 + 3x - 1 = 0$

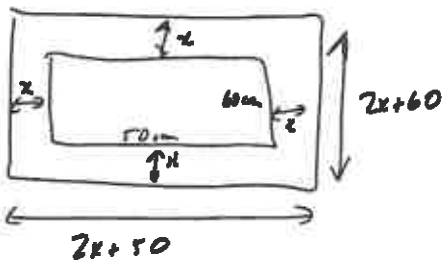
$a=2 \quad b=3 \quad c=-1$

$$x = \frac{-3 \pm \sqrt{3^2 - 4(2)(-1)}}{2(2)}$$

$$= \frac{-3 \pm \sqrt{9 + 8}}{4} \quad (17)$$

$$= \boxed{\frac{-3 \pm \sqrt{17}}{4}} \approx 0.28 \quad -1.78$$

ex. Leah wants to frame an oil original painted on canvas measuring 50 cm by 60 cm. Before framing, she places the painting on a rectangular mat so that a uniform strip of the mat shows on all sides of the painting. The area of the mat is twice the area of the painting. How wide is the strip of exposed mat showing on all sides of the painting, to the nearest tenth of a centimetre?



$$(2x+50)(2x+60) = 2(50)(60)$$

$$4x^2 + 120x + 100x + 3000 = 6000$$

$$\frac{4x^2 + 220x - 3000 = 0}{4} \quad /4$$

$$x^2 + 55x - 750 = 0$$

$$x = \frac{-55 \pm \sqrt{55^2 - 4(1)(-750)}}{2(1)}$$

$$= \frac{-55 \pm \sqrt{6025}}{2}$$

$$\approx 11.3 \quad \text{or} \quad -66.3$$

11.3 cm

Practice: pg. 254/ # 1cde, 3adf, 4de, 5, 7(use a <u>different algebraic</u> method for each), 8, 10 – 13, 21, 22