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# Unit 3: Quadratic Functions

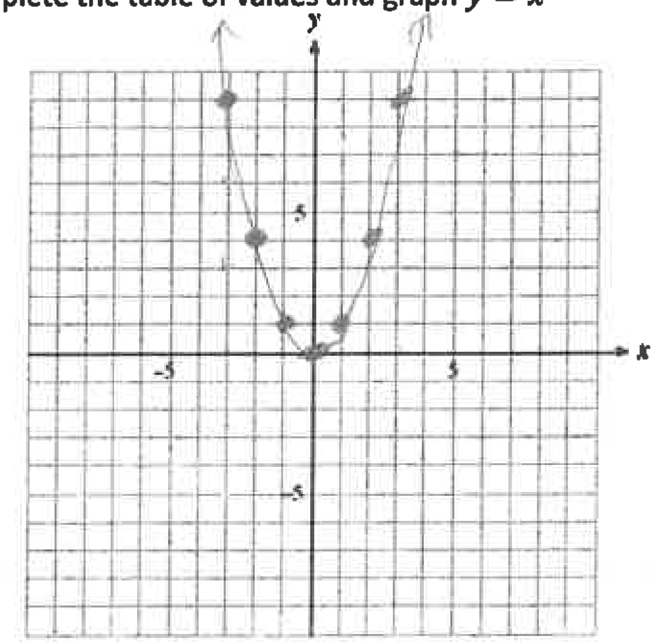
## 3.1A Quadratic Functions in Vertex Form I

**Quadratic Funtion:** A function given by a polynomial of degree 2
ex. $y = x^2$ or $f(x) = x^2$

**Parabola:** the symmetrical curve of the graph of a quadratic function

**ex.** Complete the table of values and graph $y = x^2$

| $x$ | $y$ |
|---|---|
| -3 | 9 |
| -2 | 4 |
| -1 | 1 |
| 0 | 0 |
| 1 | 1 |
| 2 | 4 |
| 3 | 9 |



$y = x^2$
$y = (-3)^2$
$y = 9$

$y = (-2)^2$
$y = 4$

**Vertex:** lowest point (opening upward) OR highest point (opening downward) of the parabola

**Minimum/Maximum Value:**
– the least value (opens up) or greatest value (opens down) of the function

**Axis of Symmetry:**
– the line that divides the graph into two congruent halves

**Vertex Form of a Quadratic Function:**
$y = a(x-p)^2 + q$ or $f(x) = a(x-p)^2 + q$

**Translation:**
moving a graph horizontally or vertically

**Vertical Translations**: $y = x^2 + q$ or $f(x) = x^2 + q$

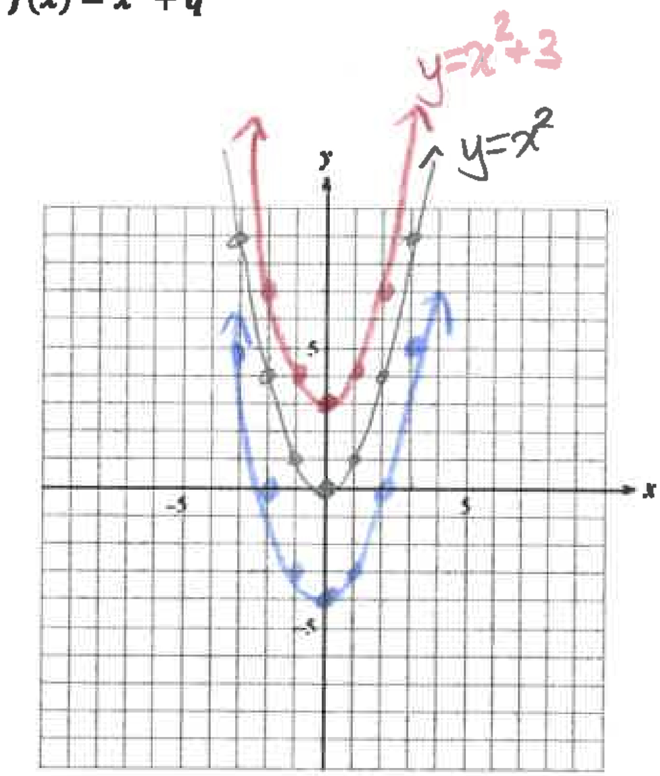
ex. Sketch:

a) $y = x^2 + 3$

| x | y |
|---|---|
| -3 | 12 |
| -2 | 7 |
| -1 | 4 |
| 0 | 3 |
| 1 | 4 |
| 2 | 7 |
| 3 | 12 |

b) $f(x) = x^2 - 4$

| x | y |
|---|---|
| -3 | 5 |
| -2 | 0 |
| -1 | -3 |
| 0 | -4 |
| 1 | -3 |
| 2 | 0 |
| 3 | 5 |




**Horizontal Translations**: $y = (x - p)^2$ or $f(x) = (x - p)^2$

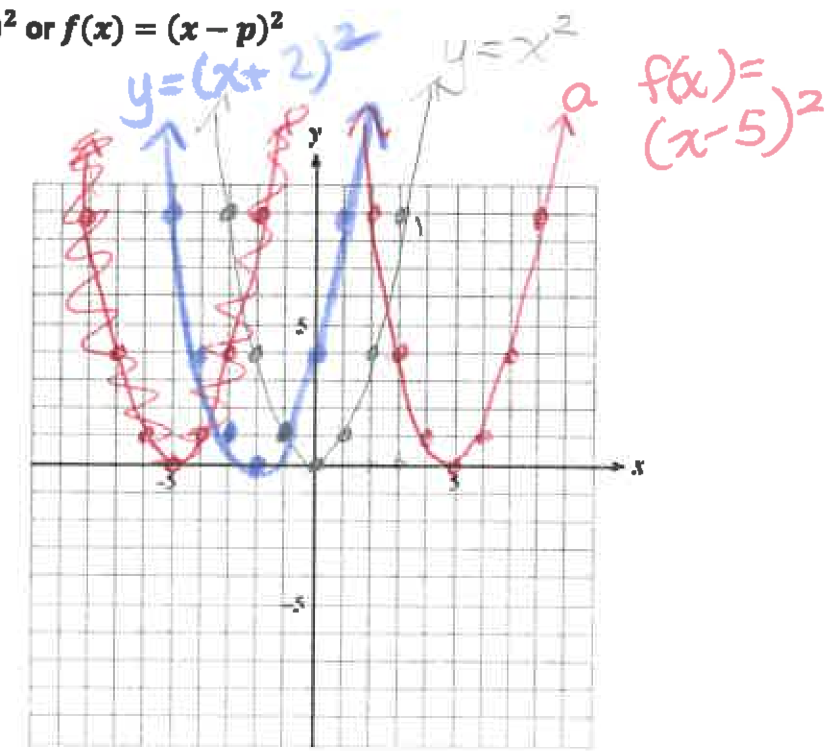
ex. Sketch:

a) $f(x) = (x - 5)^2$

| x | y |
|---|---|
| -3 | 64 |
| -2 | 49 |
| -1 | 36 |
| 0 | 25 |
| 1 | 16 |
| 2 | 9 |
| 3 | 4 |

b) $y = (x + 2)^2$

| x | y |
|---|---|
| -3 | 1 |
| -2 | 0 |
| -1 | 1 |
| 0 | 4 |
| 1 | 9 |
| 2 | 16 |
| 3 | 25 |



**Combinations:** $y = (x-p)^2 + q$ or $f(x) = (x-p)^2 + q$

vertex $(p, q)$

ex. Complete without graphing, then sketch:

a) $f(x) = (x-1)^2 - 5$

Vertex: $(1, -5)$

Axis of Symmetry: $x = 1$

Domain: $x \in \mathbb{R}$ element of, all real #'s

Range: $y \geq -5$

Max/Min: $y = -5$

b) $y = (x+3)^2 + 1$

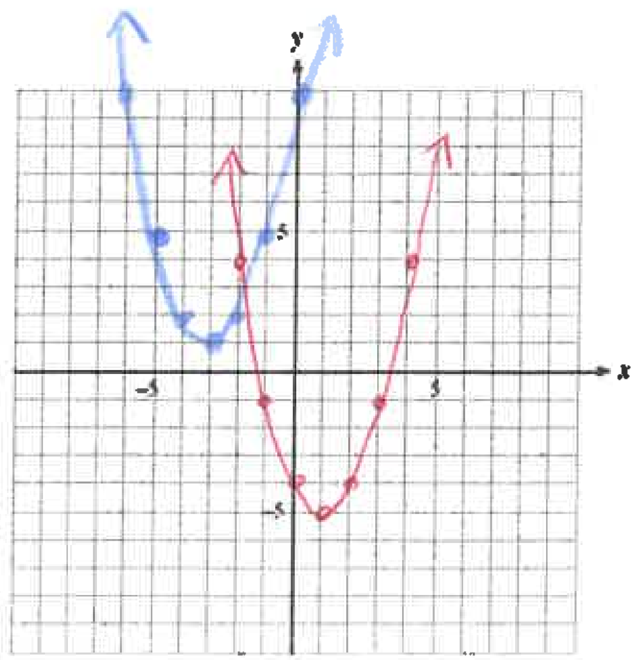
Vertex: $(-3, 1)$

Axis of Symmetry: $x = -3$

Domain: $x \in \mathbb{R}$

Range: $y \geq 1$

Max/Min: $y = 1$




Practice: pg. 157 / #2

& WS3.1A

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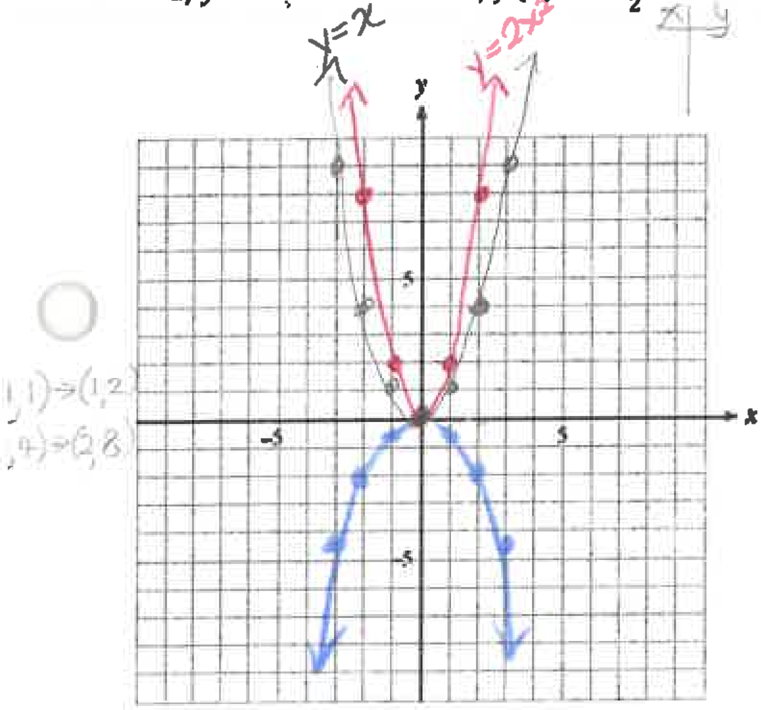
# Unit 3: Quadratic Functions

## 3.1B Quadratic Functions in Vertex Form II

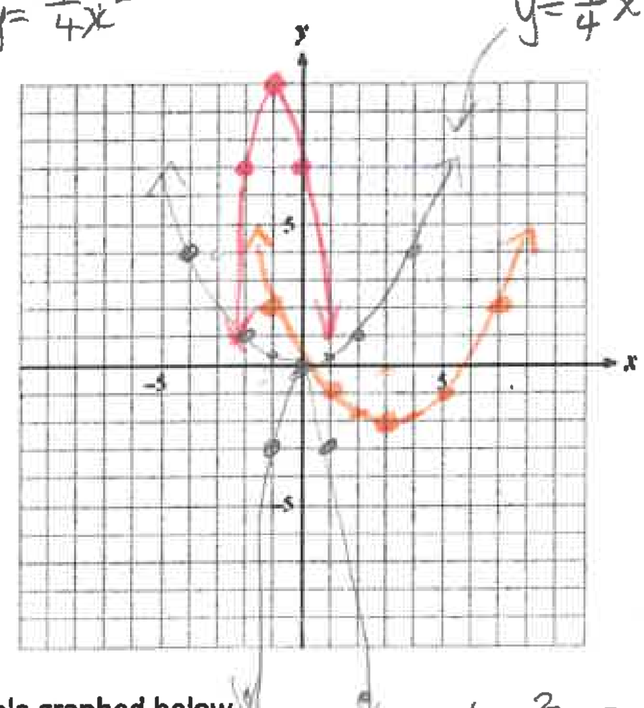
**Vertical Stretches (Expansions/Compressions) & Reflections:** $y = ax^2$ or $f(x) = ax^2$

**ex. Sketch:**

a) $y = 2x^2$ b) $f(x) = -\frac{1}{2}x^2$ c) $f(x) = \frac{1}{4}(x-3)^2 - 2$ d) $y = -3(x+1)^2 + 10$

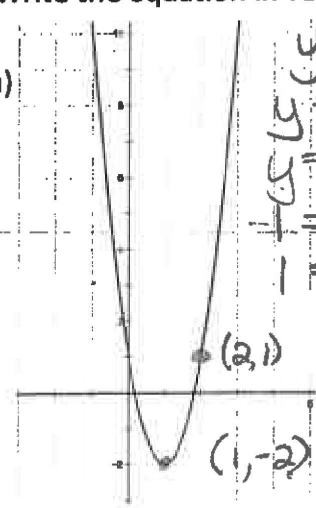



$y = \frac{1}{4}x^2$




**ex.** Write the equation in vertex form of each parabola graphed below.

a)




$y=a(x-p)^2+q$

$y=a(x-1)^2+(-2)$

$y=a(x-1)^2-2$

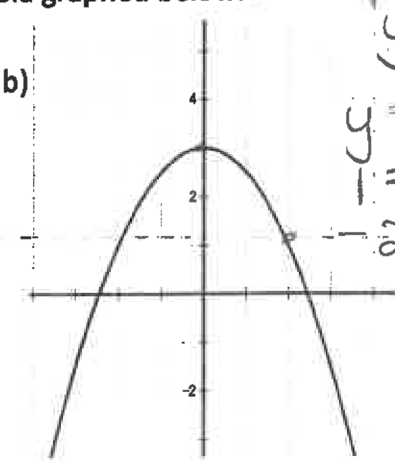
$1=a(2-1)^2-2$

$1=a-2$

$a=3$

$y=3(x-1)^2-2$

b)



$y=-\frac{1}{2}x^2+3$

$y=ax^2+3$

$1=a(2)^2+3$

$-2=4a$

$a=-\frac{1}{2}$

ex. A parabola has vertex $(\overset{x}{6}, \overset{y}{-5})$ and passes through the point $(2, -9)$. Determine its equation in vertex form.

$y = a(x-6)^2 - 5$

$-9 = a(2-6)^2 - 5$

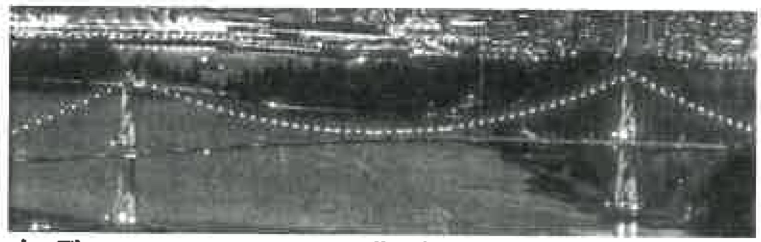
$-9 = a16 - 5$

$-4 = 16a$

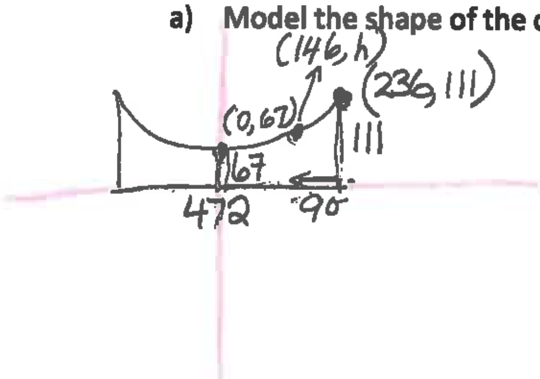
$-\frac{1}{4} = a$

$y = -\frac{1}{4}(x-6)^2 - 5$

ex. The deck of the Lions' Gate Bridge in Vancouver is suspended from two main cables attached to the tops of two supporting towers. Between the towers, the main cables take the shape of a parabola as they support the weight of the deck. The towers are 111 m tall relative to the water's surface and are 472 m apart. The lowest point of the cables is approximately 67 m above the water's surface.



a) Model the shape of the cables with a quadratic function in vertex form.




$y = ax^2 + 67$

$111 = a(236)^2 + 67$

$44 = 55696a$

$a = \frac{11}{13924}$

$y = \frac{11}{13924}x^2 + 67$

$y = 0.00079x^2 + 67$

b) Determine the height above the surface of the water of a point on the cables that is 90 m horizontally from one of the towers. Express your answer to the nearest hundredth of a meter.

$236 - 90 = 146$

$(146, y)$

$y = 0.00079(146)^2 + 67$

$y = 83.84\text{m}$

Practice: pg. 157/ # 4 (omit intercepts), 5 – 11, 14, 16 – 18, 21

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Exploration: Quadratic Functions in Standard Form

Vertex form quadratic: $f(x) = a(x - p)^2 + q$ where $a, p, q \in \mathbb{R}; a \neq 0$
Standard form quadratic: $f(x) = ax^2 + bx + c$ where $a, b, c \in \mathbb{R}; a \neq 0$

Using technology, graph the quadratic function $f(x) = -x^2 + 4x + 5$.
Describe any symmetry that the graph has.

Does the function have a maximum $y$-value? Does it have a minimum $y$-value? Explain. Use "trace" to approximate the max or min.

Using technology, graph on a Cartesian plane the functions that result from substituting the following $c$-values into the function $f(x) = -x^2 + 4x + c$.
10
0
-5
What do your graphs show about how the function $f(x) = ax^2 + bx + c$ is affected by changing the parameter $c$?

Using technology, graph on a Cartesian plane the functions that result from substituting the following $a$-values into the function $f(x) = ax^2 + 4x + 5$.
-4
-2
1
2
How is the function affected when the value of $a$ is changed?

How is the graph different when $a$ is a positive number?

Using technology, graph on a Cartesian plane the functions that result from substituting the following $b$-values into the function $f(x) = -x^2 + bx + 5$.
2
0
-2
-4
What effect does changing the value of $b$ have on the graph of the function?

Exploration: Quadratic Functions in Standard Form

Consider the vertex form quadratic $f(x) = 3(x+2)^2 - 4$.
State the values of $a, p$ & $q$

Expand $f(x) = 3(x+2)^2 - 4$ into standard form.

State the values of $a, b$ & $c$

How do the values of $a, p$ & $q$ compare to the values of $a, b$ & $c$?

Expand $f(x) = a(x-p)^2 + q$ into standard form.

Express $p$ & $q$ in terms of $a, b$ & $c$?

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# Unit 3: Quadratic Functions

## 3.3 Completing the Square

We convert a quadratic function from standard form $f(x) = ax^2 + bx + c$ to vertex form $f(x) = a(x-p)^2 + q$ using a process called **completing the square**

**ex.** Convert to vertex form: $y = x^2 + 8x + 10$

*recall algebra tile method & replicate algebraically

**Step** | **Algebra tiles** | **Algebraic**

Add & subt 16

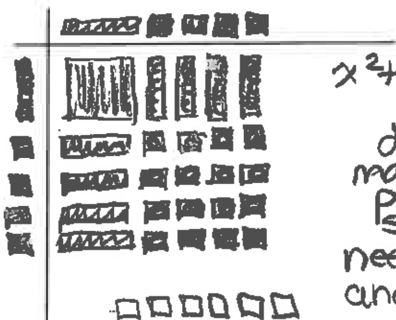
$y = (x^2 + 8x + \underline{16}) + 10 - 16$

$\left(\frac{8}{2}\right)^2$

$y = (x+4)^2 - 6$

1) Lay out trinomial
- separate constant

2) complete the square
- balance out the added values



$x^2 + 8x + 10$

doesn't make a perfect square
need to add 6 and subtract 6

3) factor the perfect square
- combine constants

$(x+4)^2 - 6$

**ex.** convert each quadratic to vertex form:

$f(x) = x^2 - 2x + 3$ Add & subt 1

$y = (x^2 - 2x + 1) + 3 - 1$

$\left(\frac{2}{2}\right)^2 = 1$

$y = (x-1)^2 + 2$

$y = x^2 + 3x + 1$

$y = \left(x^2 + 3x + \frac{9}{4}\right) + 1 - \frac{9}{4}$

$\left(\frac{3}{2}\right)^2$

$y = \left(x + \frac{3}{2}\right)^2 - \frac{5}{4}$

When completing the square, we **must** factor out leading coefficients $\neq 1$ from the $x$ terms.
Be careful with balancing the completion of the square!

**ex.** convert each quadratic to vertex form:

$y = 3x^2 - 12x + 6$

need to factor out $\neq$ in front of $x^2$

Add 12 subt 12

$3(x^2 - 4x + 4) + 6 - 12$

$\left(\frac{4}{2}\right)^2$

$y = 3(x-2)^2 - 6$

$f(x) = -x^2 + 10x - 20$

subt 25 Add 25

$y = -(x^2 - 10x + 25) - 20 + 25$

$\left(\frac{10}{2}\right)^2$

$y = -(x-5)^2 + 5$

$-4(\frac{25}{4}) = -25$

Add 25

$f(x) = -4x^2 - 20x + 1$

$y = -4(x^2 - 5x + \frac{25}{4}) + 1 + 25$

$(\frac{5}{2})^2$

$y = -4(x - \frac{5}{2})^2 + 26$

$y = 2x^2 + 7x$

$y = 2(x^2 + \frac{7}{2}x + 3.0625) - 6.125$

$(\frac{3.5}{2})^2$

$y = 2(x + 1.75)^2 - 6.125$

**ex.** The leadership class is planning a fundraising with a professional photographer taking portraits of individuals or groups. The class gets to charge and keep a session fee for each individual or group photo sessions. Last year, they charged a \$10 session fee and 400 sessions were booked. In considering what price they should charge this year, the class estimates that for every \$1 increase in the price, they expect to have 20 fewer sessions booked. Let c = # of \$1 increases

a) Write a function to model this situation.

Revenue = cost per session × # of sessions

$R(c) = (10 + c)(400 - 20c)$

$= 4000 - 200c + 400c - 20c^2$

$\boxed{R(c) = -20c^2 + 200c + 4000}$

b) What is the maximum revenue they can expect based on these estimates? What session fee will give that maximum? Need the vertex (5, 4500)

$R(c) = -20(c^2 - 10c + 25) + 4000 + 500$

$R(c) = -20(c - 5)^2 + 4500$

max revenue of \$4500 when cost is increased \$5
∴ \$15 session fee

c) How can you verify the solution?

graph

Practice: pg. 192/#

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# Exploration: Completing the Square

Note: = + (positive red tiles) = - (negative blue tiles)

**Part I** Use algebra tiles to determine each binomial square. Sketch the results.

$(x+2)^2 =$

$(x-4)^2 =$

What shape do all the tiles as a whole make in the solution layout?

Describe the solution layout as four quadrants?

Are the square quadrants always positive or negative?

**Part II** Use algebra tiles to explain why each trinomial is NOT a perfect square. Sketch the results.

$x^2 + 6x + 10$

$x^2 + 6x - 9$

$x^2 - 7x + 9$

**Part III** For each non-perfect square trinomial add-on extra zero pairs to "complete the square". Then factor the square and write each trinomial as a binomial square plus a constant. Sketch the results.

$x^2 - 10x$

$x^2 + 8x + 3$ (*pull the constant to the side)

$x^2 - 4x + 5$

$x^2 + 6x - 7$

**PART IV** Fill in the blanks to make each equation true.

| | |
|---|---|
| 1. $(x^2 + 10x + \_\_\_\_\_) = (x + 5)^2$ | 2. $(x^2 - 14x + \_\_\_\_\_) = (x - 7)^2$ |
| 3. $(x^2 - 12x\ \_\_\_\_\_) = (x\_\_\_\_\_)^2$ | 4. $(x^2 - 18x\ \_\_\_\_\_) = (x\ \_\_\_\_\_)^2$ |
| 5. $(x^2 + 100x\ \_\_\_\_\_) = (x\ \_\_\_\_\_)^2$ | 6. $(x^2 + 5x\ \_\_\_\_\_) = (x\ \_\_\_\_\_)^2$ |
| 7. $(x^2 - 11x\ \_\_\_\_\_) = (x\ \_\_\_\_\_)^2$ | 8. $(x^2 + \frac{3}{4}x\ \_\_\_\_\_) = (x\ \_\_\_\_\_)^2$ |